✎ 演習問題の別解

Chapter 2 (p.252〜)

1 (3)

$$I = 18\int_0^1 \frac{u^2}{u+\sqrt{1-u^2}}du$$

$u=\sin\theta$ とおいて $I = 18\int_0^{\frac{\pi}{2}} \frac{\sin^2\theta\cos\theta}{\sin\theta+\cos\theta}d\theta.$ さらに $\theta = \frac{\pi}{2}-\tau$ とおいて

$$I = 18\int_{\frac{\pi}{2}}^0 \frac{\cos^2\tau\sin\tau}{\sin\tau+\cos\tau}(-d\tau) = 18\int_0^{\frac{\pi}{2}} \frac{\sin\theta\cos^2\theta}{\sin\theta+\cos\theta}d\theta$$

$$\therefore \quad I+I = 18\int_0^{\frac{\pi}{2}} \frac{\sin\theta^2\cos\theta+\sin\theta\cos^2\theta}{\sin\theta+\cos\theta}d\theta = 18\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta d\theta = 18\left[\frac{1}{2}\sin^2\theta\right]_0^{\frac{\pi}{2}}$$

$$\therefore \quad I = \frac{9}{2}(\sin^2\frac{\pi}{2}-0^2) = \frac{9}{2}$$

Chapter 3 (p.255〜)

1 (2)

$$\cosh(cx-4c^3t) = \frac{e^{cx-4c^3t}+e^{-cx+4c^3t}}{2} = g(x,t),$$

$$\sinh(cx-4c^3t) = \frac{e^{cx-4c^3t}-e^{-cx+4c^3t}}{2} = h(x,t)$$

とおくと，$g_x = ch$, $g_t = -4c^3h$, $h_x = cg$ となる．

$$\begin{aligned}
v =& \frac{-2c^2}{\cosh^2(cx-4c^3t)} = -2c^2g^{-2},\\
v_t =& -2c^2(-2g^{-3}g_t) = 4c^2g^{-3}(-4c^3h) = -16c^5g^{-3}h,\\
v_x =& -2c^2(-2g^{-3}g_x) = 4c^2g^{-3}(ch) = 4c^3g^{-3}h,\\
v_{xx} =& 4c^3(-3g^{-4}ch\cdot h + g^{-3}\cdot cg) = 4c^4g^{-4}(g^2-3h^2),\\
v_{xxx} =& 4c^4\{-4g^{-5}ch(g^2-3h^2)+g^{-4}(2gch-6hcg)\} = 16c^5g^{-5}h(-2g^2+3h^2)
\end{aligned}$$

$$\begin{aligned}
\therefore \quad v_t -& 6ccv_x + x_{xxx}\\
=& -16c^5g^{-3}h - 6(-2c^2g^{-2})\cdot 4c^3g^{-3}h + 16c^5g^{-5}h(-2g^2+3h^2)\\
=& 16c^5g^{-5}h\{3-3(g^2-h^2)\}
\end{aligned}$$

$g^2-h^2=1$ だから $v_t - 6ccv_x + x_{xxx} = 0.$